Faculté de Droit & des Sciences Économiques — Université de Limoges

ANNÉE UNIVERSITAIRE 2021-2022

Session 1 **Semestre 4**

Licence Economie-Gestion – 2e année

Matière : Statistiques et probabilités **Durée :** 2 heures
Enseignant : Vincent Jalby

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Problème

En 2019, on évalue à 1602 € la dépense annuelle moyenne d'énergie par les ménages français pour leur logement (source : `http://www.statistiques.developpement-durable.gouv.fr`) avec un écart-type de 450 €. (Dans la suite, on notera simplement **dépense énergétique**.)

Partie I (30 min, 6 points)

Afin d'actualiser ces données pour l'année 2021, une collectivité territoriale a effectué un sondage sur un échantillon aléatoire de ménages. Les résultats sont disponibles dans la sortie JAMOVI (`https://www.jamovi.org`) ci-dessous.

Descriptives

				90% Confidence Interval				
	N	Mean	SE	Lower	Upper	SD	Minimum	Maximum
Depense	256	1655.793	29.051	1608.009	1703.577	464.811	185.000	2918.000

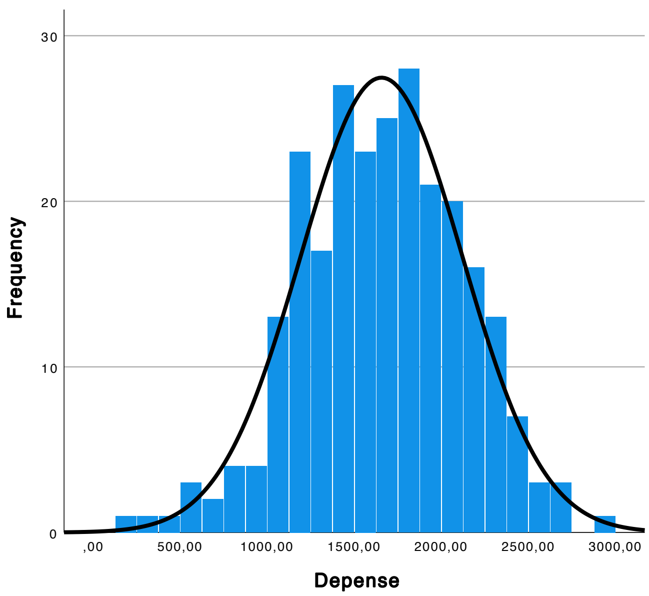
1) Décrire le modèle statistique (population, variable, loi, paramètre) pour modéliser ce sondage.

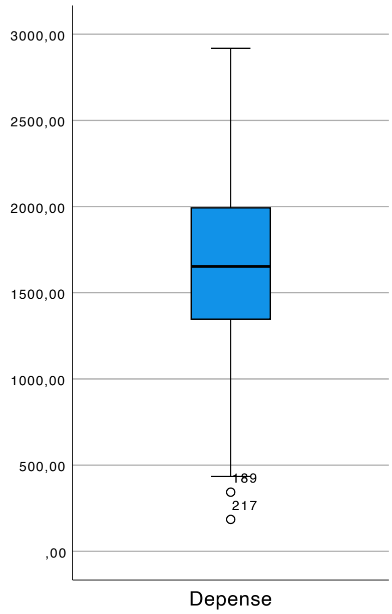
2) Donner une estimation de la dépense énergétique moyenne, en la justifiant théoriquement.

3) Donner un intervalle de confiance de la dépense énergétique moyenne à 95 %.

4) Commenter la sortie JAMOVI ci-dessus.

5) Discuter la normalité des données en commentant les sorties SPSS ci-dessous :





Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Depense	,030	256	,200*	,995	256	,488

Partie II (30 min, 5 points)

On souhaite d'abord déterminer si les variations de dépenses énergétiques ont changé en 2021 (par rapport à 2019).

1) Poser clairement les hypothèses du test.

2) Construire un test avec un risque de première espèce de 10 % permettant de vérifier l'hypothèse ci-dessus.

3) Quelle est votre conclusion ?

4) Commenter la sortie Stata suivante :

```
Stata. sdtest Depense == 450

One-sample test of variance
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
 Depense |     256    1655.793    29.05067    464.8107    1598.583    1713.003
------------------------------------------------------------------------------
    sd = sd(Depense)                                        c = chi2 = 272.0617
Ho: sd = 450                                     degrees of freedom =      255

    Ha: sd < 450                 Ha: sd != 450                 Ha: sd > 450
  Pr(C < c) = 0.7789         2*Pr(C > c) = 0.4421           Pr(C > c) = 0.2211
```

Partie III (30 min, 5 points)

On souhaite à présent tester si la dépense énergétique moyenne a augmenté de 2019 à 2021.

1) Poser clairement les hypothèses du test puis effectuer le test en prenant un risque de 5 %.

2) Quelle est votre conclusion ? Quel risque prenez-vous ?

3) Calculer la probabilité critique associée à ce test.

4) Quelle aurait été la conclusion si on avait pris un risque de 1 % ?

5) Commenter la sortie R suivante :

```
R> t.test(depense, mu=1602)

        One Sample t-test

data:  depense
t = 1.8517, df = 255, p-value = 0.06523
alternative hypothesis: true mean is not equal to 1602
95 percent confidence interval:
 1598.583 1713.003
sample estimates:
mean of x
 1655.793
```

Partie IV (30 min, 4 points)

L'échantillon précédent est composé de ménages résidant en habitat collectif (immeuble) et individuel (maison). On souhaite déterminer si la dépense énergétique moyenne est différente selon le type d'habitation.

1) Présenter la procédure statistique adaptée.

2) Commenter, exhaustivement, les sorties SAS ci-dessous :

Type	Method	N	Mean	Std Dev	Std Err	Mean	95% CL Mean		Std Dev
Individuel		119	1658.8	438.5	40.1990	1658.8	1579.2	1738.4	438.5
Collectif		137	1562.3	450.4	38.4807	1562.3	1486.2	1638.4	450.4
Diff (1-2)	Pooled		96.4889	444.9	55.7533	96.4889	-13.3088	206.3	444.9
Diff (1-2)	Satterthwaite		96.4889		55.6482	96.4889	-13.1087	206.1	

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	136	118	1.05	0.7675

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	254	1.73	0.0847
Satterthwaite	Unequal	250.7	1.73	0.0842

Récapitulatif des lois discrètes

Loi	Notation	Support	Loi	Espérance	Variance
Bernoulli	$\mathcal{B}(1,p)$	$X(\Omega)=\{0,1\}$	$P(X=0)=q \quad P(X=1)=p$	$\mathrm{E}(X)=p$	$\mathrm{Var}(X)=pq$
Binomiale	$\mathcal{B}(n,p)$	$X(\Omega)=\{0,\dots,n\}$	$P(X=k)=\binom{n}{k}p^kq^{n-k}$	$\mathrm{E}(X)=np$	$\mathrm{Var}(X)=npq$
Hypergéométrique	$\mathcal{H}(N,n,p)$	$X(\Omega)\subset\{0,\dots,n\}$	$P(X=k)=\dfrac{\binom{Np}{k}\times\binom{Nq}{n-k}}{\binom{N}{n}}$	$\mathrm{E}(X)=np$	$\mathrm{Var}(X)=npq\dfrac{N-n}{N-1}$
Géométrique	$\mathcal{G}(p)$	$X(\Omega)=\mathbb{N}^*$	$P(X=k)=pq^{k-1}$	$\mathrm{E}(X)=\dfrac{1}{p}$	$\mathrm{Var}(X)=\dfrac{q}{p^2}$
Pascal	Pascal(r,p)	$X(\Omega)=\{r,r+1,\dots\}$	$P(X=k)=\binom{k-1}{r-1}p^rq^{k-r}$	$\mathrm{E}(X)=\dfrac{r}{p}$	$\mathrm{Var}(X)=\dfrac{rq}{p^2}$
Poisson	$\mathcal{P}(\lambda)$	$X(\Omega)=\mathbb{N}$	$P(X=k)=\mathrm{e}^{-\lambda}\dfrac{\lambda^k}{k!}$	$\mathrm{E}(X)=\lambda$	$\mathrm{Var}(X)=\lambda$

$p\in[0,1] \qquad q=1-p \qquad n,N,r\in\mathbb{N}^* \qquad \lambda>0$

Récapitulatif des lois continues

Loi	Notation	Support	Loi/Densité	Espérance	Variance
Uniforme	$\mathcal{U}(a,b)$	$X(\Omega)=[a,b]$	$f_X(x)=\dfrac{1}{b-a}$ si $x\in[a,b]$	$\mathrm{E}(X)=\dfrac{a+b}{2}$	$\mathrm{Var}(X)=\dfrac{(b-a)^2}{12}$
Exponentielle	$\mathcal{E}(\lambda)$ $\mathrm{Exp}(\lambda)$	$X(\Omega)=[0,+\infty[$	$f_X(x)=\lambda\mathrm{e}^{-\lambda x}$ si $x\geq 0$	$\mathrm{E}(X)=\dfrac{1}{\lambda}$	$\mathrm{Var}(X)=\dfrac{1}{\lambda^2}$
Normale	$\mathcal{N}(\mu,\sigma)$	$X(\Omega)=\mathbb{R}$	$f_X(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\mathrm{E}(X)=\mu$	$\mathrm{Var}(X)=\sigma^2$
Normale standard (Z)	$\mathcal{N}(0,1)$	$Z(\Omega)=\mathbb{R}$	$f_Z(z)=\dfrac{1}{\sqrt{2\pi}}\mathrm{e}^{-z^2/2}$	$\mathrm{E}(Z)=0$	$\mathrm{Var}(Z)=1$
Khi-deux (K^2)	$\chi^2(n)$	$K^2(\Omega)=[0,+\infty[$	$K^2=\sum_{i=1}^{n}Z_i^2$ où $Z_i\hookrightarrow\mathcal{N}(0,1)$ ind.	$\mathrm{E}(K^2)=n$	$\mathrm{Var}(K^2)=2n$
Student (T)	$St(n)$	$T(\Omega)=\mathbb{R}$	$T=\dfrac{Z}{\sqrt{K^2/n}}$ où $\begin{cases}Z\hookrightarrow\mathcal{N}(0,1)\\K^2\hookrightarrow\chi^2(n)\end{cases}$ ind.	$\mathrm{E}(T)=0$	$\mathrm{Var}(T)=\dfrac{n}{n-2}$
Fisher (F)	$F(n_1,n_2)$	$F(\Omega)=[0,+\infty[$	$F=\dfrac{K_1^2/n_1}{K_2^2/n_2}$ où $\begin{cases}K_1^2\hookrightarrow\chi^2(n_1)\\K_2^2\hookrightarrow\chi^2(n_2)\end{cases}$ ind.	$\mathrm{E}(F)=\dfrac{n_2}{n_2-2}$	$\mathrm{Var}(F)=\dfrac{2n_2^2(n_1+n_2-2)}{n_1(n_2-2)^2(n_2-4)}$

$a,b\in\mathbb{R} \qquad a<b \qquad \lambda>0 \qquad \mu\in\mathbb{R} \qquad \sigma>0 \qquad n,n_1,n_2\in\mathbb{N}^*$

Fonction de répartition de la loi normale standard

$$P(\mathcal{N}(0,1) \leq z)$$

Exemple : $P\big(\mathcal{N}(0,1) \leq 1.33\big) = 0.9082$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Quantiles de la loi de Student

$$P(St(n) \leq t_\alpha) = \alpha$$

Exemple : $P\big(St(11) \leq 2.201\big) = 0.975$

n\α	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	0.975	0.99	0.995	0.999	0.9995
1	0.158	0.325	0.510	0.727	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.142	0.289	0.445	0.617	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.137	0.277	0.424	0.584	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.134	0.271	0.414	0.569	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.132	0.267	0.408	0.559	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.131	0.265	0.404	0.553	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.130	0.263	0.402	0.549	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.130	0.262	0.399	0.546	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.129	0.261	0.398	0.543	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.129	0.260	0.397	0.542	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.129	0.260	0.396	0.540	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.128	0.259	0.395	0.539	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.128	0.259	0.394	0.538	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.128	0.258	0.393	0.537	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.128	0.258	0.393	0.536	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.128	0.258	0.392	0.535	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.128	0.257	0.392	0.534	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.127	0.257	0.392	0.534	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.127	0.257	0.391	0.533	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.127	0.257	0.391	0.533	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.127	0.257	0.391	0.532	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.127	0.256	0.390	0.532	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.127	0.256	0.390	0.532	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.127	0.256	0.390	0.531	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.127	0.256	0.389	0.531	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.127	0.256	0.389	0.530	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.126	0.255	0.388	0.529	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.126	0.255	0.388	0.528	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.126	0.254	0.387	0.527	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
70	0.126	0.254	0.387	0.527	0.678	0.847	1.044	1.294	1.667	1.994	2.381	2.648	3.211	3.435
80	0.126	0.254	0.387	0.526	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
90	0.126	0.254	0.387	0.526	0.677	0.846	1.042	1.291	1.662	1.987	2.368	2.632	3.183	3.402
100	0.126	0.254	0.386	0.526	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
110	0.126	0.254	0.386	0.526	0.677	0.845	1.041	1.289	1.659	1.982	2.361	2.621	3.166	3.381
120	0.126	0.254	0.386	0.526	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
130	0.126	0.254	0.386	0.526	0.676	0.844	1.041	1.288	1.657	1.978	2.355	2.614	3.154	3.367
140	0.126	0.254	0.386	0.526	0.676	0.844	1.040	1.288	1.656	1.977	2.353	2.611	3.149	3.361
150	0.126	0.254	0.386	0.526	0.676	0.844	1.040	1.287	1.655	1.976	2.351	2.609	3.145	3.357
∞	0.126	0.253	0.385	0.524	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Quantiles de la loi du Khi-deux

$$P(\chi^2(n) \leq k_\alpha^2) = \alpha$$

Exemple : $P\left(\chi^2(15) \leq 22.31\right) = 0.90$

$n\backslash\alpha$	**0.005**	**0.01**	**0.025**	**0.05**	**0.10**	**0.25**	**0.4**	**0.5**	**0.6**	**0.75**	**0.90**	**0.95**	**0.975**	**0.99**	**0.995**
1	0.000	0.000	0.001	0.004	0.016	0.102	0.275	0.455	0.708	1.323	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	0.575	1.022	1.386	1.833	2.773	4.605	5.991	7.378	9.210	10.60
3	0.072	0.115	0.216	0.352	0.584	1.213	1.869	2.366	2.946	4.108	6.251	7.815	9.348	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	1.923	2.753	3.357	4.045	5.385	7.779	9.488	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.610	2.675	3.655	4.351	5.132	6.626	9.236	11.07	12.83	15.09	16.75
6	0.676	0.872	1.237	1.635	2.204	3.455	4.570	5.348	6.211	7.841	10.64	12.59	14.45	16.81	18.55
7	0.989	1.239	1.690	2.167	2.833	4.255	5.493	6.346	7.283	9.037	12.02	14.07	16.01	18.48	20.28
8	1.344	1.646	2.180	2.733	3.490	5.071	6.423	7.344	8.351	10.22	13.36	15.51	17.53	20.09	21.95
9	1.735	2.088	2.700	3.325	4.168	5.899	7.357	8.343	9.414	11.39	14.68	16.92	19.02	21.67	23.59
10	2.156	2.558	3.247	3.940	4.865	6.737	8.295	9.342	10.47	12.55	15.99	18.31	20.48	23.21	25.19
11	2.603	3.053	3.816	4.575	5.578	7.584	9.237	10.34	11.53	13.70	17.28	19.68	21.92	24.72	26.76
12	3.074	3.571	4.404	5.226	6.304	8.438	10.18	11.34	12.58	14.85	18.55	21.03	23.34	26.22	28.30
13	3.565	4.107	5.009	5.892	7.042	9.299	11.13	12.34	13.64	15.98	19.81	22.36	24.74	27.69	29.82
14	4.075	4.660	5.629	6.571	7.790	10.17	12.08	13.34	14.69	17.12	21.06	23.68	26.12	29.14	31.32
15	4.601	5.229	6.262	7.261	8.547	11.04	13.03	14.34	15.73	18.25	22.31	25.00	27.49	30.58	32.80
16	5.142	5.812	6.908	7.962	9.312	11.91	13.98	15.34	16.78	19.37	23.54	26.30	28.85	32.00	34.27
17	5.697	6.408	7.564	8.672	10.09	12.79	14.94	16.34	17.82	20.49	24.77	27.59	30.19	33.41	35.72
18	6.265	7.015	8.231	9.390	10.86	13.68	15.89	17.34	18.87	21.60	25.99	28.87	31.53	34.81	37.16
19	6.844	7.633	8.907	10.12	11.65	14.56	16.85	18.34	19.91	22.72	27.20	30.14	32.85	36.19	38.58
20	7.434	8.260	9.591	10.85	12.44	15.45	17.81	19.34	20.95	23.83	28.41	31.41	34.17	37.57	40.00
21	8.034	8.897	10.28	11.59	13.24	16.34	18.77	20.34	21.99	24.93	29.62	32.67	35.48	38.93	41.40
22	8.643	9.542	10.98	12.34	14.04	17.24	19.73	21.34	23.03	26.04	30.81	33.92	36.78	40.29	42.80
23	9.260	10.20	11.69	13.09	14.85	18.14	20.69	22.34	24.07	27.14	32.01	35.17	38.08	41.64	44.18
24	9.886	10.86	12.40	13.85	15.66	19.04	21.65	23.34	25.11	28.24	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	19.94	22.62	24.34	26.14	29.34	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	20.84	23.58	25.34	27.18	30.43	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	21.75	24.54	26.34	28.21	31.53	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	22.66	25.51	27.34	29.25	32.62	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	23.57	26.48	28.34	30.28	33.71	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	24.48	27.44	29.34	31.32	34.80	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	33.66	37.13	39.34	41.62	45.62	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	42.94	46.86	49.33	51.89	56.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	52.29	56.62	59.33	62.13	66.98	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	61.70	66.40	69.33	72.36	77.58	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	71.14	76.19	79.33	82.57	88.13	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	80.62	85.99	89.33	92.76	98.65	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	90.13	95.81	99.33	102.9	109.1	118.5	124.3	129.6	135.8	140.2
110	75.55	78.46	82.87	86.79	91.47	99.67	105.6	109.3	113.1	119.6	129.4	135.5	140.9	147.4	151.9
120	83.85	86.92	91.57	95.70	100.6	109.2	115.5	119.3	123.3	130.1	140.2	146.6	152.2	159.0	163.6
130	92.22	95.45	100.3	104.7	109.8	118.8	125.3	129.3	133.4	140.5	151.0	157.6	163.5	170.4	175.3
140	100.7	104.0	109.1	113.7	119.0	128.4	135.1	139.3	143.6	150.9	161.8	168.6	174.6	181.8	186.8
150	109.1	112.7	118.0	122.7	128.3	138.0	145.0	149.3	153.8	161.3	172.6	179.6	185.8	193.2	198.4
200	152.2	156.4	162.7	168.3	174.8	186.2	194.3	199.3	204.4	213.1	226.0	234.0	241.1	249.4	255.3